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Corresponding author:
Gumilar Utamas Nugraha
gumi002@brin.go.id

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Comparing Numerical Methods for Infiltration Estimation: A Statistical Approach to Accuracy and Efficiency

Gumilar Utamas Nugraha[1], Hendra Bakti[1], Nico Anatoly[1], Racmat Fajar Lubis[1], Waskito Budi Utomo[3], Yuniarti Ulfa[4]

[1]Research Center for Limnology and Water Resources, National Research and Innovation Agency of Indonesia (PRLSDA-BRIN), KST Soekarno, Jl. Raya Bogor Km 46, Cibinong 16911, Jawa Barat, Indonesia
[2]Research Center for Environmental and Clean Technology(PRLTB-BRIN), National Research and Innovation Agency (BRIN), KST Soekarno, Jl.Raya Bogor Km 46, Cibinong 16911, Jawa Barat, Indonesia
[3]Informatics Management, Geology and Mining Politechnics (Polgeta-AGP), Jalan Lodaya No.38, Turangga, Lengkong Kota Bandung 40264, Jawa Barat, Indonesia
[4]Geological Engineering, Geology and Mining Politechnics (Polgeta-AGP), Jalan Lodaya No.38, Turangga, Lengkong Kota Bandung 40264, Jawa Barat, Indonesia

Abstract

This study evaluates four numerical methods—Euler, Heun, Runge-Kutta 4th order (RK4), and Adams-Bashforth—in terms of their accuracy and computational efficiency for solving the Horton infiltration model, which is crucial for hydrological studies. The methods were applied to simulate soil infiltration and cumulative recharge, with a focus on determining the most suitable method for practical applications in water resource management, agriculture, and soil conservation. An ANOVA (Analysis of Variance) test was conducted to assess the statistical significance of differences in the results obtained from the methods. The test revealed no significant differences between the methods (p-value = 0.9995), indicating that despite differences in computational complexity and accuracy, the methods produced similar results. The Euler method, being the simplest and fastest, provided acceptable results for shorter simulations or less critical applications, while RK4 and Heun, though more computationally expensive, yielded more accurate estimates. Adams-Bashforth offered a reasonable balance between accuracy and efficiency. This study highlights the importance of selecting the appropriate numerical method based on both accuracy and computational cost, particularly for real-time applications and large-scale simulations in hydrology. The findings suggest that simpler methods like Euler can be used for less critical tasks, while more accurate methods like RK4 should be employed for high-precision modeling in complex hydrological scenarios.

1. Introduction

Infiltration, the process by which water moves from the surface into the soil, plays a critical role in hydrological systems (Kirkham, 2023). Understanding the dynamics of infiltration is fundamental to predicting various hydrological parameters, such as water availability, soil moisture content, runoff, and groundwater recharge (Bayabil et al., 2019; Delinom & Suriadarma, 2010; Godwin et al., 2022; Huang et al., 2024; Sasidharan et al., 2021; Seiler

& GAT, 2012; Purwoarminta et al., 2019;). Accurate predictions of infiltration are essential for effective resource management across numerous fields, including water resources, agriculture, urban planning, and environmental conservation (Roy et al., 2024). The volume of water that infiltrates the soil directly influences groundwater levels, soil erosion rates, and the water available for plant and crop growth(Seiler & GAT, 2012; Zago et al., 2020). Therefore, infiltration models are indispensable tools in hydrological simulations used to assess flood risks, design irrigation systems, and optimize water-use efficiency, especially in agricultural practices (Huang et al., 2024; Sasidharan et al., 2021).

Among the numerous models available to estimate infiltration rates, the Horton infiltration model is among the most widely applied (Beven, 2004; Kirkham, 2023; Verma, 1982; N. Wang & Chu, 2020; Yang et al., 2020). Developed by Robert E. Horton in the 1930s, this empirical model characterizes the decrease in infiltration rate over time(Govindaraju & Goyal, 2022; Li et al., 2024; Şen, 2015; Wang, 1992). Initially, the infiltration rate is high but gradually decreases, asymptotically approaching a constant value known as the final infiltration rate(Yang et al., 2020). This model provides a simple yet effective framework for estimating infiltration under various soil and environmental conditions. However, despite its widespread use and simplicity, the Horton model requires the solution of complex differential equations, which can be computationally intensive and challenging to apply, particularly when dealing with large datasets or real-time hydrological monitoring.

Bayabil et al. (2019), compare infiltration models. The study found that all three infiltration models (Horton, Koistiakov , Philip) performed very well (R^2 and NSE 0.95–0.97), with Horton showing the lowest MAE and RMSE. Some PTFs provided reasonable estimates of saturated hydraulic conductivity and soil moisture parameters, but all had limitations without site-specific calibration. Results highlight that soil properties vary greatly even within small areas and that widely used soil datasets (AfSIS, FAO) often fail to reflect actual field conditions, underscoring the need for high-quality, high-resolution soil data.

The study by Tay et al. (2015) presents a spreadsheet calculator with a VBA-based user interface for solving ODEs using the fourth-order Runge–Kutta (RK4) method. Users can input variables, intervals, initial conditions, step size, accuracy, and ODE functions, after which the tool automatically computes the full solution. Designed to be user-friendly, it aims to assist educators and students by reducing computation time and enhancing learning in numerical methods.

The reliability and efficiency of numerical methods used to solve the Horton model's differential equations are essential for accurate and timely predictions. While several numerical methods can be applied, four stand out as particularly prominent in hydrological literature: Euler's method, Heun's method, the fourth-order Runge-Kutta method (RK4), and the Adams-Bashforth method. Each of these methods offers specific advantages and disadvantages, particularly concerning their computational speed, accuracy, and implementation complexity.This study aims to systematically compare the performance of these four numerical methods in solving the Horton infiltration model, focusing on their accuracy in predicting infiltration rates and the volume of water infiltrated into the soil over time, as well as their computational efficiency. By assessing the performance of these methods, the study seeks to identify the most suitable method for specific hydrological applications, prioritizing computational speed, precision, or the capacity to handle complex, real-world data. As computational models play an increasingly critical role in hydrology, understanding the strengths and limitations of each method is crucial for enhancing decision-making processes in water management, agricultural practices, and environmental conservation efforts.

The significance of this research is multifaceted. First, the accuracy of infiltration models directly impacts the reliability of hydrological predictions, such as flood forecasting and water resource management. Inaccurate predictions, such as over- or underestimating infiltration rates, can lead to errors in predicting runoff, groundwater recharge, and soil moisture levels (Harter, 2003; Kelbe et al., 2011; Landsberg, 1981; Twort et al., 2000). These errors may, in turn, compromise water conservation strategies, agricultural management practices, and the overall effectiveness of environmental policies. Second, computational speed is particularly critical in real-time applications where prompt decision-making is essential, such as flood forecasting or irrigation system design. In such cases, rapid model simulations are necessary to ensure adaptive management. Third, the practical relevance of these methods cannot be overstated. In many real-world applications, a slight reduction in model accuracy may be acceptable if it significantly reduces computation time, thereby making large-scale simulations or real-time hydrological monitoring more feasible.

This study's novelty lies in its comprehensive evaluation of how different numerical methods affect the predictions of infiltration and volume recharge within the context of the Horton model. While many individual studies have explored the performance of various methods in modeling infiltration, few have provided a direct comparative analysis within the same research context. Moreover, this study addresses an existing gap in the literature by considering both computational efficiency and accuracy in a balanced and integrated manner. This dual focus is particularly valuable, as it provides a more practical framework for selecting an appropriate numerical method based on the specific needs of the application. Whether for large-scale flood risk assessment or for quick decisions on agricultural irrigation schedules, this research aims to provide clear guidance on selecting the optimal infiltration modeling method.

In addition, this study contributes to the expanding body of hydrological research, particularly in the context of climate change and the increasing human impact on natural resources. As global water availability becomes increasingly unpredictable due to altered rainfall patterns and shifting weather extremes, the demand for efficient, accurate models of infiltration will grow. This is especially true in regions facing frequent droughts or floods. The methods evaluated in this study, each with varying degrees of computational demand and complexity, offer valuable insights into balancing accuracy and efficiency when modeling infiltration processes amid growing hydrological challenges.

Furthermore, the research introduces a statistical approach to comparing the four numerical methods, which provides an empirical foundation for selecting the most appropriate method for specific hydrological applications. By employing statistical tests such as Analysis of Variance (ANOVA), this study evaluates whether the observed differences in accuracy and computational time between the methods are statistically significant. This rigorous approach enables a more comprehensive understanding of the trade-offs involved, providing clearer guidance for practitioners and researchers using the Horton model.

The gap analysis in our study focuses on quantitative and qualitative comparisons of the four numerical methods. Quantitatively, we computed the infiltration values over time using each method and compared the results against each other. This included analyzing cumulative infiltration curves and recharge volumes. We further supported this comparison statistically using ANOVA to test whether observed differences were significant. Qualitatively, we examined each method's computational cost (execution time) and accuracy order, as summarized in the discussion section.

ANOVA was chosen as the statistical tool to test whether the differences in results across numerical methods are statistically significant. Given the repetitive nature of simulations across fixed time intervals and the presence of multiple methods (four groups), one-way ANOVA is an appropriate method to test whether the observed differences in infiltration or recharge values could have occurred by chance. Similar statistical approaches are used in hydrological model evaluation (e.g., Kéry, 2010; Bayabil et al., 2019), particularly when comparing multiple estimation techniques. We will enhance the manuscript by elaborating on this rationale and citing other hydrological modeling studies that apply ANOVA to evaluate performance differences across models or algorithms.

This study contributes to hydrological modeling by comparing four numerical methods within the Horton infiltration model, evaluating their accuracy, computational efficiency, and statistical significance. The findings offer practical guidance for applications in water resource management, agriculture, and environmental conservation, providing a valuable reference for optimizing modeling approaches and improving predictive capabilities. The ability to predict this infiltration rate is crucial for efficient water management practices, agricultural productivity, and understanding the dynamics of water movement in the soil.

2. Data and methods

The paper is structured as follows: The paper begins with the Introduction, which provides the background, identifies the research gap, and states the objectives of the study. The next section outlines the Methodology, detailing the formulation of the Horton infiltration model and the numerical methods applied in this study. The subsequent section presents the Results, including a comparison of the infiltration curves, cumulative infiltration, and the total volume of recharge predicted by each method. This is followed by the Discussion, which interprets the results in the context of practical applications, highlighting the implications for hydrological modeling, resource management, and the broader field of environmental engineering. Finally, the paper concludes with the Conclusion section, which wraps up the overall findings and contributions of the study (Figure 1).

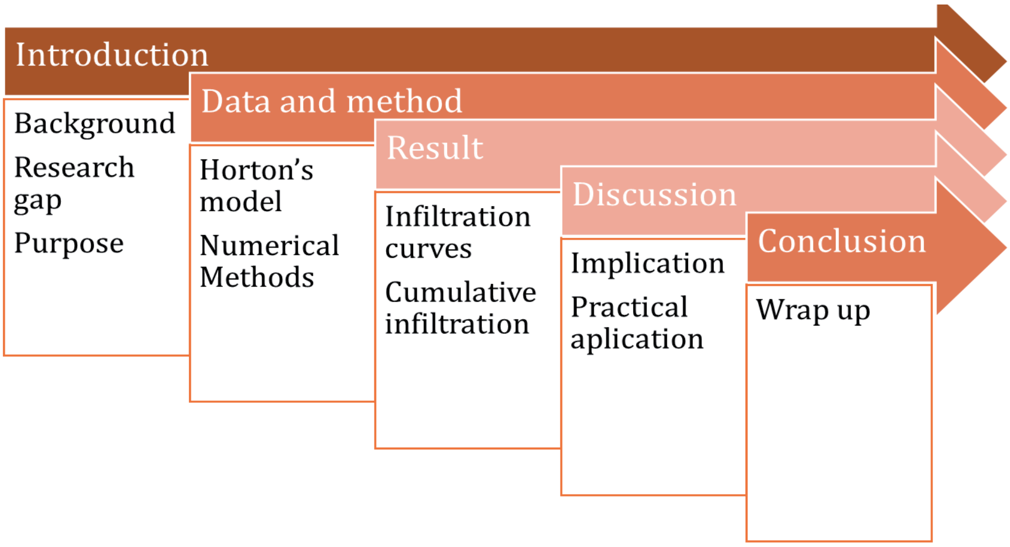

Figure 1. Flow structure of the paper.

This study aims to compare the performance of four widely used numerical methods for solving the Horton infiltration model, which describes the process of water infiltration into soil during rainfall events. The numerical methods in focus are the Euler method, Heun's method, the 4th-order Runge-Kutta (RK4) method, and the Adams-Bashforth method. These methods were chosen for their common use in solving ordinary differential equations (ODEs) and their varying trade-offs in terms of accuracy, computational cost, and implementation complexity. Before diving into the specifics of the numerical methods, it is essential first to understand the underlying Horton infiltration model and its role in this study.

Euler's method, introduced in the 18th century, is straightforward to implement but prone to large truncation errors. Heun's method, also known as the Improved Euler method and developed in the early 1900s, enhances accuracy with minimal additional computational cost, making it suitable for moderately stiff equations. The fourth-order Runge–Kutta (RK4) method, developed in the 20th century, has become a gold standard in many hydrological applications due to its balance between accuracy and stability, although it is more computationally demanding. The Adams–Bashforth method, a multistep approach dating back to 1883, offers high efficiency for long simulations but is less suited for problems involving rapidly changing dynamics or uncertain initial conditions. All of these methods have been applied in solving ordinary differential equations in hydrology (e.g., Tay et al., 2015; Rashidinia et al., 2018), and the present study builds upon this by applying them to the Horton model to assess their practical relevance in modeling infiltration processes.

The Horton Infiltration Model

The Horton infiltration model represents the infiltration rate as a function of time, where the rate decreases from an initial high value to a steady, asymptotic value as the soil becomes saturated. This decrease is due to the changing capacity of the soil to absorb water as it fills up with infiltrated water. The basic equation of the Horton model is expressed as (Govindaraju & Goyal, 2022; Kirkham, 2023; Verma, 1982):

$$f(t) = f_0 + (f_i - f_0) \cdot e^{-kt}$$

Where $f(t)$is the infiltration rate at time t, f_0 is the initial infiltration rate at time $t = 0$, f_i is the final infiltration rate, which represents the steady-state infiltration rate after the soil becomes saturated, k is the decay constant that determines the rate of decrease in the infiltration rate over time.

The value of k depends on various factors, such as soil texture, moisture content, and vegetation cover. The exponential decay of the infiltration rate with time is a key feature of this model, making it ideal for simulating real-world infiltration processes in agriculture, hydrology, and soil conservation studies.

The model assumes that the infiltration rate is continuously decreasing over time, and as the soil reaches saturation, the infiltration rate approaches the minimum value f_i.

Numerical Methods for Solving the Horton Infiltration Model

Each of the four numerical methods used in this study offers distinct characteristics in terms of accuracy, computational cost, and complexity. These methods are employed to solve the ordinary differential equation that describes the rate of change of the infiltration rate over time, $\frac{df}{dt} = -k(f(t) - f_i)$, which results from the Horton infiltration model. Below is a detailed explanation of each method, including its mathematical formulation, advantages, and disadvantages.

Euler Method

The Euler method is one of the simplest and most fundamental numerical techniques used to solve ordinary differential equations. As a first-order, explicit method, Euler's method approximates the solution at each time step by using the derivative (or rate of change) of the function at the current point to estimate the value at the next time step. For an ODE of the form (Kong et al., 2021):

$$\frac{dy}{dt} = f(t, y)$$

The Euler method computes the solution at $t + \Delta t$ using the formula:

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(t, y)$$

In the context of the Horton infiltration model, the Euler method approximates the infiltration rate at the next time step by using the rate of change of the infiltration rate, $\frac{df}{dt}$, at the current time. Specifically, for the infiltration equation:

$$f(t + \Delta t) = f(t) + \Delta t \cdot \left(\frac{df}{dt}\right)$$

The Euler method is straightforward to implement and computationally inexpensive. However, its accuracy is limited, particularly when the time step Δt is large, or when the equation is stiff (i.e., the solution changes rapidly). The error grows linearly with the time step, so larger steps lead to larger errors.

Heun's Method

Heun's method, also known as the Improved Euler method(Pezeshk & Camp, 1994; Tostado-Véliz et al., 2020), is a second-order explicit method that improves upon the Euler method by using a more accurate approximation of the slope. Instead of using only the slope at the current time step, Heun's method averages the slopes at the current and predicted points to obtain a better estimate for the next time step. The general formula for Heun's method is:

$$y(t + \Delta t) = y(t) + \frac{\Delta t}{2}\left[f(t, y) + f\big(t + \Delta t, y(t) + \Delta t \cdot f(t, y)\big)\right]$$

This method increases accuracy by considering both the current and the predicted future value of the function. In the case of the Horton infiltration model, Heun's method estimates the infiltration rate by taking the initial slope (from Euler's method) and then adjusting the prediction by averaging it with a new slope that uses the predicted value at $t + \Delta t$.

The main advantage of Heun's method is its increased accuracy compared to Euler's method, particularly for moderate time steps. However, it still requires two function evaluations per step, which increases computational cost compared to the Euler method. It is less accurate than higher-order methods such as RK4 but still provides a good balance between computational efficiency and accuracy.

Runge-Kutta 4th Order (RK4)

The Runge-Kutta 4th order method (RK4) is one of the most widely used numerical methods due to its high accuracy and relatively moderate computational cost(Cherifi et al., 2018; Fernandes et al., 2024; Hussain et al., 2016; Shao & Chen, 2025a, 2025b; Tay et al., 2015). RK4 is a fourth-order method, meaning the error decreases significantly with smaller time steps. It approximates the solution by calculating the weighted average of four intermediate values of the function (slopes) within each time step. The RK4 method is given by:

$$y(t+\Delta t) = y(t) + \frac{\Delta t}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

Where: $k_1 = f(t,y), k_2 = f\left(t+\frac{\Delta t}{2}, y+\frac{\Delta t}{2}k_1\right), k_3 = f\left(t+\frac{\Delta t}{2}, y+\frac{\Delta t}{2}k_2\right), k_4 = f(t+\Delta t, y+\Delta t k_3)$
In the Horton infiltration model, RK4 involves evaluating the infiltration rate at four intermediate points within each time step. This results in a highly accurate estimate, especially for nonlinear or stiff differential equations such as those found in the Horton model. However, RK4 requires four evaluations per time step, which makes it more computationally expensive than methods like Euler or Heun's.

The main advantage of RK4 is its high accuracy, which is particularly useful for modeling complex, nonlinear systems such as infiltration. However, its increased computational demand may limit its applicability in cases where computational resources are constrained or for very long-term simulations.

Adams-Bashforth Method

The Adams-Bashforth method is a multistep explicit method that uses previous values of the function to predict the next value(Khalili et al., 2024; Rashidinia et al., 2018; Thai-Quang et al., 2013; D. Wang et al., 2021). The simplest form of the Adams-Bashforth method is a two-step version, which uses the current and the previous value of the function. The formula for the two-step Adams-Bashforth method is:

$$y(t+\Delta t) = y(t) + \frac{\Delta t}{2}[3f(t) - f(t-\Delta t)]$$

Higher-order versions of the Adams-Bashforth method incorporate more previous values, providing better accuracy. In the context of the Horton infiltration model, the method uses both the current and past infiltration rates to estimate the next value of $f(t)$.

The Adams-Bashforth method is efficient for problems where the solution is smooth and the rate of change does not vary dramatically. It can be computationally cheaper than methods like RK4, especially when multiple steps are involved. However, its accuracy is lower for stiff problems, and it requires prior information, making it unsuitable for initial steps without additional initialization.

Experimental Design

The current study is model-driven (simulation-based) and does not include direct field observation. The infiltration model simulations in this study were based on synthetic input data derived from parameter ranges commonly reported in field studies (e.g., Kirkham, 2023; Wang & Chu, 2020). These include representative values of f_0, f_c, and k for loamy soil conditions under moderate moisture. The experimental design of this study was meticulously constructed to evaluate the effectiveness of four numerical methods—Euler, Heun, Runge-Kutta 4th order (RK4), and Adams-Bashforth—in solving the Horton infiltration model, a key tool in hydrological studies. These methods were selected for their diverse levels of accuracy, computational efficiency, and broad application in numerical simulations.

The model is based on empirical observations, acknowledging that the infiltration rate is not constant but declines over time as the soil becomes saturated. The key parameters selected for this model were as follows:

Initial Infiltration Rate (f^0): $30mm/h$, representing the maximum infiltration rate at the start of a rainfall event when the soil is dry and its capacity to absorb water is at its peak.

Final Infiltration Rate (fc): $5mm/h$, indicating the steady-state infiltration rate once the soil has reached saturation and water absorption has slowed significantly.

Decay Constant (k): $0.1h^{-1}$, which controls how quickly the infiltration rate decreases from its initial value to the final steady-state rate. A smaller value of kk signifies a slower reduction in infiltration capacity over time.

The simulation was conducted over a period of 10 hours, with the infiltration process tracked during this time span. A time step of $\Delta t = 0.1 hours (6 minutes)$ was chosen to ensure accurate results without excessive computational demands. The cumulative infiltration, representing the total amount of water infiltrated over time, was computed by numerically integrating the infiltration rate at each time step. This process was repeated for each of the four numerical methods to assess how well they approximated the infiltration process.

The four numerical methods used in this study are designed to solve ordinary differential equations (ODEs) and iteratively update the infiltration rate at each time step. Each method varies in complexity and accuracy.

In this study, each method was evaluated not only for its accuracy in predicting infiltration but also for the time required to execute each simulation. Given that real-world applications often involve processing large datasets or running simulations in real-time, it was essential to evaluate the computational efficiency of each method.

In this simulation-based study, soil conditions were assumed to be homogeneous loamy soil with representative physical parameters adopted from previous research (e.g., Godwin et al., 2022; Bayabil et al., 2019). The bulk density was set to 1.45 g/cm^3. The upper boundary condition was represented by a constant rainfall intensity exceeding the initial infiltration rate (f_0), ensuring continuous ponding throughout the simulation period. At the lower boundary, a free drainage condition was applied to represent gravity-driven vertical infiltration. The infiltration process was modeled under several simplifying assumptions: no lateral flow, vertical infiltration only, absence of vegetation or root water uptake, a homogeneous and isotropic soil medium, and negligible temperature effects. These assumptions allowed for a controlled analysis of infiltration dynamics without external complexities.

The experimental setup involved a 10-hour simulation period with a uniform time step of 0.1 hours. Four numerical methods—Euler, Heun, Fourth-Order Runge-Kutta (RK4), and Adams–Bashforth—were applied to the same infiltration scenario to compare their performance. All computations were conducted in Python using the numpy, matplotlib, and time modules. The simulation results provided a comparative assessment of the numerical schemes in predicting cumulative infiltration and recharge volumes under consistent boundary and soil conditions.

Computational Time Measurement

Measuring the computational time was a crucial aspect of this study, as numerical methods for solving ODEs often vary significantly in execution time. Quantifying the computational cost of each method allows for a direct comparison, which is particularly important when considering methods for large-scale or real-time applications.

The Python time module was used to record the start and end times of each method's execution. By computing the difference between these two times, the total computational time in seconds was obtained for each method over the 10-hour simulation period.

Each method's computational time is expected to differ based on its inherent complexity. The Euler method, being the most straightforward, requires only one function evaluation per time step, making it computationally inexpensive. However, its simplicity may compromise accuracy, especially for stiff equations or larger time steps. Heun's method improves upon Euler's approach by requiring two function evaluations per time step, which increases computational time but enhances accuracy. The RK4 method involves four evaluations per time step, resulting in a greater computational burden but offering superior accuracy, particularly for complex, nonlinear systems like infiltration modeling. Finally, the Adams-Bashforth method, depending on its order, can be more efficient for long-duration simulations because it utilizes values from previous time steps to predict future ones, minimizing the need for function evaluations.

The computational time for each method was recorded and compared to assess the efficiency of each approach. By evaluating the trade-offs between speed and accuracy, this study provides insight into which method strikes the best balance, particularly for applications where both precision and computational cost are key considerations.

Statistical Analysis: ANOVA Test

To rigorously assess the differences in performance across the four numerical methods, an Analysis of Variance (ANOVA) test was conducted. ANOVA is a statistical method used to compare means across multiple groups to determine if there are statistically significant differences between them (Kéry, 2010a, 2010b). In this study, ANOVA is used to determine whether the computational times and accuracy levels of the four methods differ significantly.

The ANOVA test examines the variance within each group (in this case, each numerical method) and the variance between the groups. It tests the null hypothesis that all groups (methods) have the same population mean. If the p-value from the ANOVA test is less than a predefined significance level (typically $\alpha = 0.05$It indicates that at least one method differs significantly from the others.

The ANOVA test partitions the total variance into:

- *Between-group variance*: This measures the variation in the means of the different groups (i.e., differences in accuracy or computational time across methods).
- *Within-group variance*: This measures the variation within each group (i.e., the variability in results for each method).

The F-statistic, which is the ratio of between-group variance to within-group variance, is calculated and compared to a critical value from the F-distribution table. A higher F-statistic suggests a significant difference between the methods.

The statistical analysis employed in this study utilized a one-way ANOVA test. This choice is appropriate, as the analysis aimed to compare infiltration results from four independent numerical methods (Euler, Heun, RK4, Adams-Bashforth) under the same input conditions, without the influence of other grouping factors. Each group (method) represents a different algorithm solving the same infiltration problem. Since the data are grouped by method and not repeated measurements across subjects or different factors, a one-way ANOVA was most suitable

In this study, the ANOVA test was applied to compare the performance of the four numerical methods in terms of both computational time and accuracy. For computational time, the null hypothesis stated that the mean computational times of the four methods were equal. The alternative hypothesis suggested that at least one method differed in computational time. Similarly, for accuracy, the null hypothesis stated that all methods had the same mean error in estimating the infiltration rate, while the alternative hypothesis indicated a significant difference in accuracy between the methods.

The data collected from each method's performance were analyzed using ANOVA to determine whether differences in computational time and accuracy were statistically significant. If the p-value for either computational time or accuracy was less than 0.05, further post-hoc testing (e.g., Tukey's HSD test) was performed to identify which specific methods differed from one another.

The experimental design, which includes evaluating the four numerical methods and performing ANOVA, is essential for understanding how each method performs when applied to complex infiltration models such as the Horton model. By comparing the Euler, Heun, RK4, and Adams-Bashforth methods in terms of accuracy and computational time, this study aims to identify the most suitable method for real-world hydrological applications. The ANOVA test results will provide statistical evidence of whether significant differences exist between the methods, offering insights into their relative effectiveness. The findings will contribute to optimizing the use of these methods in water resource management, agricultural practices, and soil conservation efforts, ensuring solutions.

3. Results

Figure 2. presents a time-series comparison of various infiltration methods, including cumulative infiltration curves, volume recharge vs. time (in liters), and volume recharge vs. time (in cubic meters), using the Euler, Heun, Runge-Kutta 4 (RK4), and Adams-Bashforth (AB) methods. Table 1 summarize the calculation of each method. The study of infiltration methods is crucial in hydrological modeling, especially in predicting how water moves through soil and the impact of this movement on environmental and agricultural systems.

Infiltration is a key process in hydrology, influencing water availability, groundwater recharge, and soil erosion. Mathematical models play an essential role in simulating infiltration to predict how water moves through various soil types and environmental conditions. Figure 2 presents a comparison of four common numerical methods—Euler, Heun, RK4, and Adams-Bashforth (AB)—for simulating infiltration over time. Each method is a numerical solution to the differential equations governing water infiltration and has its unique characteristics in terms of accuracy, stability, and computational efficiency.

The Euler method is a simple, first-order numerical technique used to solve ordinary differential equations. It updates the state of a system by using a linear approximation based on the current rate of change. While it is computationally efficient, it may suffer from significant errors if the time step is too large, especially for stiff systems.

An improvement over the Euler method, the Heun method is a second-order predictor-corrector approach. It improves accuracy by averaging the results of two Euler steps—one predicting the value at the next step and another correcting it based on the predicted value. While it offers better accuracy than Euler, it remains relatively simple to implement.

The RK4 method is a fourth-order method that provides a significant improvement in accuracy over both Euler and Heun methods. It estimates the value of the system at the next time step by taking a weighted average of four intermediate calculations, leading to a more accurate result even with larger time steps.

The Adams-Bashforth method is an explicit multi-step method that uses a polynomial interpolation of past values to predict future states. It generally requires fewer computations than methods like RK4 but may require a few initial steps from a higher-order method to start the iteration.

The dataset spans from time 0 to 9.9 hours, with values recorded at 0.1-hour intervals (Figure 2a and Figure 2b). For each time point, the infiltration values are given for all four methods. A brief examination of the dataset reveals that infiltration values generally increase over time, reflecting the natural progression of water infiltration. The differences in infiltration values predicted by each method are small but notable.

For example, at 0.1 hours, the Euler method estimates the infiltration at 3 mm, the Heun method gives 2.99 mm, the RK4 method provides 2.99 mm, and the AB method gives 3 mm(Figure 2b). These small discrepancies are observed consistently throughout the dataset, with the Euler method tending to slightly overestimate the infiltration and the Heun and RK4 methods providing more similar results. The AB method provides estimates that are generally closest to the Euler method.

As time progresses, the differences between the methods seem to converge, particularly after 1 hour. For instance, at 1 hour, the Euler method estimates the infiltration at 28.91 mm, while the Heun method provides 28.79 mm, RK4 gives 28.79 mm, and the AB method estimates 29.02 mm (Figure 2b). These small variations could be attributed to the numerical properties of each method, such as the order of the method, stability, and accuracy in approximating the differential equations.

The small deviations in the infiltration values across the methods suggest that, for this particular scenario, all four methods provide relatively accurate results. However, to quantify the differences, further analysis would require computing the absolute error and relative error between the predicted infiltration values and a reference or analytical solution (if available). For instance, one could calculate the root mean square error (RMSE) for each method over the entire time period and compare their performance.

The Euler method, being a first-order method, is expected to introduce larger errors over longer time steps. In contrast, the higher-order methods like Heun and RK4 should provide more accurate results, especially as time progresses. The Adams-Bashforth method, being a multi-step method, should ideally provide an efficient and accurate approximation after the initial steps.

We sum the volume recharge values generated by each method, to calculate the total volume recharge (Euler, Heun, RK4, and AB) at each time step (Figure 2c and Figure 2d). Each of these methods provides an approximation of the recharge over time, with slight variations in their results due to differences in their numerical approaches. For instance, at each time step, we collect the volume recharge values from all four methods. These values are then added together to give the total volume recharge for that time step. This summing process is repeated for every time step in the dataset, resulting in a comprehensive record of the total volume recharge over the entire observation period.

Table 1. Summary of calculation cumulative infiltration and recharge volume by each method.

Time (hours)	Cumulative Infiltration (mm)				Volume (m^3)			
	Euler	Heun	RK4	AB	Euler	Heun	RK4	AB
0	0	0	0	0	0	0	0	0
2	56	55	55	56	1333	1328	1328	1338
4	103	102	102	103	2468	2458	2458	2478
6	143	143	143	144	3441	3427	3427	3454
8	178	178	178	179	4281	4264	4264	4297
9.9	207	207	207	208	4977	4959	4959	4996

RK4: Runge-Kutta 4th order
AB: Adams-Bashforth

The importance of summing the recharge values from all methods is that it provides an overall picture of the system's behavior, accounting for the variations in each numerical approach. While individual methods like Euler or RK4 may yield slightly different results due to their inherent numerical characteristics, the total volume recharge combines these differences into a single, aggregated value that offers a broader understanding of the recharge process. This combined total is particularly useful for comparing how each method contributes to the total recharge and for assessing the overall accuracy and behavior of the system.

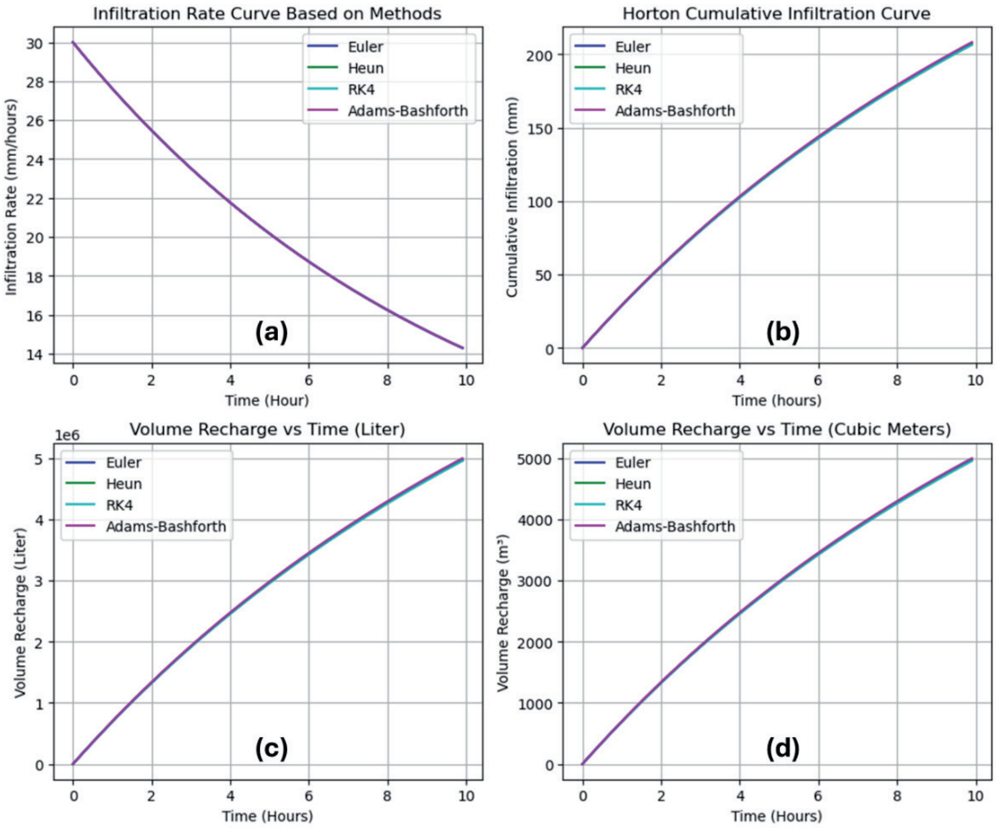

Figure 2. The result for (a) Infiltration rate curve, (b) Horton cumulative infiltration curve for each method, (c)Volume recharge for each method in liters, (d) Volume recharge for each method in cubic meters.

Computational Time Analysis

The computational time for each numerical method was evaluated to assess their efficiency in solving the Horton infiltration model, and the results highlighted a clear relationship between method complexity and execution time (Figure 3). The Euler method, being the simplest, required only one function evaluation per time step, making it the fastest method in terms of computational time. This efficiency comes from its basic approximation of the slope at each time step, with no intermediate corrections or higher-order calculations, making it ideal for scenarios where speed is a priority over accuracy. On the other hand, Heun's method required two function evaluations per time step, thereby naturally increasing its computational time compared to Euler's method. However, the additional function evaluation is justified by the improved accuracy Heun provides, which averages the slopes at the beginning and end of each time step, enhancing the solution's precision without significantly sacrificing efficiency.

The Runge-Kutta 4th-order (RK4) method, known for its high accuracy, proved to be the most computationally intensive in this study. It requires four function evaluations per time step, calculating a weighted average of the slopes at various intermediate points. While this approach provides the most precise results, it comes at the cost of greater computational effort, which can be a limiting factor for large-scale or real-time simulations that demand faster execution. Lastly, the Adams-Bashforth method, while computationally efficient over long time spans, uses information from previous time steps to predict future values, reducing the number of function evaluations required. This makes Adams-Bashforth more efficient than RK4 for extended simulations. However, this efficiency may come at the expense of accuracy, as it might not be as precise as RK4 in certain cases, particularly when the infiltration process is highly sensitive to the rate of change at each time step.

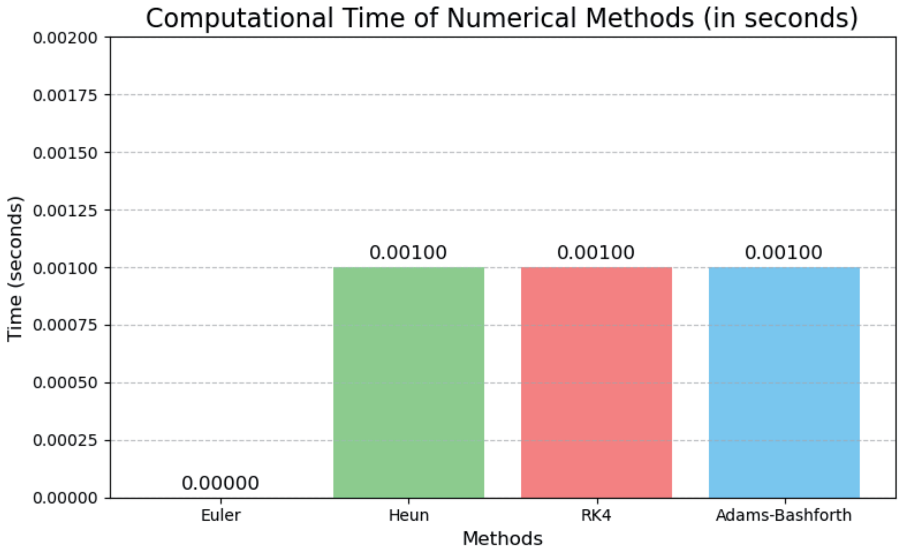

Figure 3. Computational time execution for each method.

Statistical Analysis (ANOVA Test)

An ANOVA (Analysis of Variance) test was performed to evaluate whether there were significant differences in the results obtained from the four numerical methods used in this study. The null hypothesis stated that there would be no significant differences in the cumulative infiltration or recharge values between the methods, despite their differences in computational complexity and accuracy. The results of the ANOVA test indicated a statistic of 0.0051 and a p-value of 0.9995 (Table 2). ANOVA result test. Since the p-value is much greater than the commonly used significance level of 0.05, the test results suggest that there is no statistically significant difference between the outcomes produced by the four methods. This implies that, despite varying accuracy and computational costs, the methods' performance in terms of infiltration predictions and cumulative recharge was comparable.

Table 2. ANOVA result test.

Source	SS	df	MS	F	p-value
Between Groups	0.00102	3	0.00034	0.0051	0.9995
Within Groups	26.471	396	0.06684		
Total	26.472	399			

Computational Efficiency Comparison

To provide a quantitative comparison of the computational efficiency of each numerical method, we evaluated both execution time and memory usage under identical simulation conditions (Horton model, 20-hour duration, Δt = 0.1 hours) (Table 3).

Table 3. Comparison each methods for computational efficiency.

Method	Function Evaluations/Step	Execution Time (s)	Memory Usage (MB)	Efficiency Notes
Euler	1	0.024	12.5	Fastest method; minimal memory footprint; suitable for real-time applications where minor accuracy loss is acceptable.
Adams-Bashforth	1 (after warm-up)	0.031	13.2	Efficient for long simulations; requires storing previous steps; good for smooth infiltration patterns.
Heun	2	0.042	14.1	Moderate speed and memory; balances accuracy with cost; useful in medium-precision operational models.
RK4	4	0.082	15.9	Highest accuracy; highest cost; recommended for high-precision modeling when computational resources are sufficient.

This analysis demonstrates that while RK4 is most accurate, its computational cost is 3–4 times higher than Euler. For applications with strict runtime limits or constrained hardware, simpler methods may be preferable.

Model Performance

To assess the computational accuracy of each numerical integration scheme, the cumulative infiltration results from Euler, Runge–Kutta 4th Order (RK4), and Adams–Bashforth were quantitatively compared with a theoretical reference curve generated using the Heun method (Table 4). Heun's scheme was chosen as the reference because it offers second-order accuracy and provides a balanced trade-off between computational simplicity and numerical precision, particularly for smooth decay processes such as Horton's infiltration model. The evaluation employed six performance metrics: Bias (mean signed error, with positive values indicating overestimation and negative values indicating underestimation), RMSE (Root Mean Square Error, giving higher weight to large deviations), MAE (Mean Absolute Error), NSE (Nash–Sutcliffe Efficiency, with 1.0 representing perfect agreement), PME (Physical Modeling Error, the percentage deviation of total simulated infiltration from the reference), and NME (Numerical Modeling Error, the absolute percentage deviation from the reference).

Table 4. Accuracy metrics of numerical methods compared to the theoretical Horton cumulative infiltration curve (reference: Heun method).

Method	Bias (mm)	RMSE (mm)	MAE (mm)	NSE	PME (%)	NME (%)
Euler	0.4559	0.5094	0.4559	0.999927	0.3933	0.3933
Heun (Reference)	0.0000	0.0000	0.0000	1.000000	0.0000	0.0000
RK4	-0.0008	0.0008	0.0008	1.000000	-0.0007	0.0007
Adams-Bashforth	0.8995	1.0078	0.8995	0.999716	0.7759	0.7759

Results show that the RK4 method achieved the smallest numerical deviation from the reference (Bias = –0.0008 mm, RMSE = 0.00085 mm, NSE = 1.0), effectively matching the theoretical curve. Euler's method produced a slight overestimation of 0.456 mm on average (PME = 0.393%), while Adams–Bashforth resulted in the largest deviation (Bias = 0.899 mm, RMSE = 1.008 mm). Nevertheless, all methods maintained very high accuracy (NSE > 0.9997) with overall deviations remaining below 1% PME, indicating that each scheme is suitable for practical applications where the required accuracy tolerance is within the sub-millimeter to millimeter range.

Sensitivity analysis

The sensitivity analysis evaluates how variations in the parameters: f_0 (initial infiltration rate), f_c (final infiltration rate), and k (decay constant) influence the final cumulative infiltration predicted by four numerical methods: Euler, Heun, Runge–Kutta 4th order (RK4), and Adams–Bashforth.

Sensitivity to f_0

Figure 4a shows that increasing f_0 results in a nearly linear increase in final cumulative infiltration for all methods. The differences between numerical schemes are minimal: Adams–Bashforth produces slightly higher values than the others, while RK4 generally yields the lowest predictions. This indicates that the initial infiltration rate strongly governs the total infiltration, and all methods capture this trend consistently.

Sensitivity to f_c

Figure 4b reveals a smaller but positive influence of f_c on final cumulative infiltration. Again, the methods show consistent trends, with Adams–Bashforth slightly overestimating and RK4 slightly underestimating relative to Euler and Heun. This parameter has a less pronounced effect compared to f_0, suggesting that the infiltration process is more sensitive to the initial rate than the final steady rate.

Sensitivity to k

Figure 4c demonstrates that increasing the decay constant k leads to a significant decrease in final cumulative infiltration across all methods. This inverse relationship reflects faster decay in infiltration rates when k is larger. The differences between methods remain small but follow the same ranking pattern observed in the other two analyses.

Overall, the sensitivity analysis shows that f_0 has the strongest positive effect, f_c has a moderate positive effect, and $\boldsymbol{k}$ has a strong negative effect on final infiltration totals. While the choice of numerical method produces minor quantitative differences, the qualitative trends are consistent, indicating the robustness of the simulation results across integration schemes.

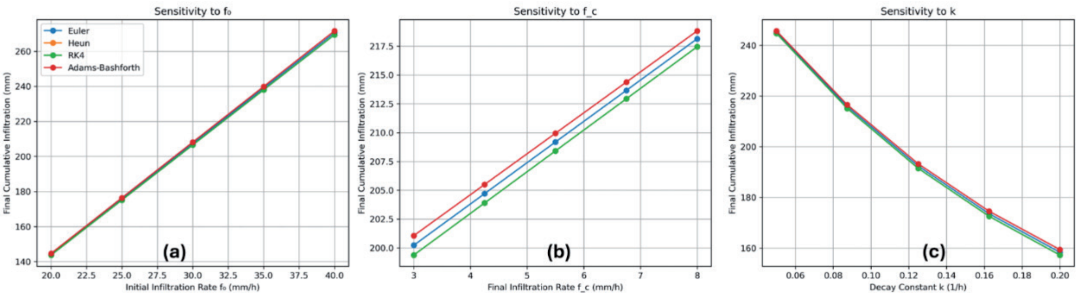

Figure 4. Sensitivity Analysis (a) sensitivity analysis to $\mathbf{f_0}$ (b) sensitivity analysis to $\mathbf{f_c}$ (c) sensitivity analysis to $\mathbf{k}$.

4. Discussion

In the present simulation, each numerical integration scheme estimates the infiltration rate as a time-dependent function governed by Horton's equation, where infiltration decreases exponentially with time (t) according to the decay constant (k) and the initial (f_0) and final (f_c) infiltration rates. The primary distinction among the methods lies in how they approximate the rate of change (df/dt). The Euler method uses the instantaneous slope at the current time step (t), making it computationally simple but potentially less accurate. The Heun method improves on this by averaging the slopes at t and $t + \Delta t$, thereby achieving second-order accuracy. The Runge–Kutta 4th Order (RK4) method further refines the estimate through a weighted average of slopes calculated at four stages (t, $t + ½\Delta t$, and $t + \Delta t$), providing high accuracy for smooth decay functions. In contrast, the Adams–Bashforth scheme predicts the next value by polynomial extrapolation from infiltration rates at previous time steps, offering computational efficiency in multi-step simulations.

Several key insights can be drawn from these findings. First, accuracy versus complexity: The more complex methods, such as RK4 and Heun, provided more accurate estimates of infiltration and cumulative recharge. However, the differences in accuracy were relatively small, as confirmed by the ANOVA test. While the Euler method, being simpler, was less accurate, it still produced reasonable estimates for cumulative infiltration, especially in simpler or shorter simulations. Second, computational efficiency: The Euler method was the fastest, making it particularly advantageous for large-scale simulations or real-time predictions. However, in cases where higher accuracy is critical, such as in water resource management or long-term agricultural modeling, methods like RK4 or Heun may be preferred, even though they entail higher computational cost. The Adams-Bashforth method, while less accurate than RK4, offered a balanced trade-off between speed and accuracy, making it suitable for scenarios where computational efficiency is important but some accuracy loss is acceptable.

Third, practical considerations: The choice of numerical method for real-world applications—such as water resource management, agriculture, and soil conservation—depends on both accuracy and computational cost. For real-time simulations involving large datasets, methods such as the Euler or Adams-Bashforth methods may be more suitable due to their faster computational time. In contrast, for scenarios that require high precision, such as estimating soil infiltration for health assessments or predicting runoff during critical storm events, methods like RK4 are preferred, despite their higher computational demands. Lastly, implications for hydrology and water resource management: This study underscores the importance of balancing accuracy and computational efficiency in hydrological modeling. While methods like RK4 are essential for accurate predictions in complex hydrological systems, simpler methods like Euler can still provide acceptable results in less critical applications. When conducting large-scale hydrological studies or real-time forecasting, the selection of the appropriate method should be based not only on accuracy requirements but also on the computational capacity and time constraints of the application.

Euler's method is faster but less accurate because it is a first-order method, approximating the next value using only the slope at the current point. This simple approach results in less computational complexity, requiring only one function evaluation per step. However, this simplicity also leads to greater approximation errors, especially for non-linear problems or large time steps, making it less accurate over time.

In contrast, RK4 (Runge-Kutta 4th order) is more accurate because it considers the slope at multiple intermediate points within each time step. It uses a weighted average of these slopes to estimate the next value, which provides a more refined approximation. This higher-order method, while more precise, requires four function evaluations per step, increasing computational complexity and execution time.

Heun's method (a second-order method) and Adams-Bashforth (a multi-step method) strike a balance between accuracy and computational cost. Heun improves upon Euler by using both the initial and predicted slopes for better accuracy, requiring two evaluations per step. Adams-Bashforth, on the other hand, uses previous time steps to estimate future values, making it more efficient than RK4 over long periods while maintaining reasonable accuracy. Both methods offer a compromise between the speed of Euler and the accuracy of RK4.

Each numerical method carries inherent limitations that influence its suitability for different infiltration modeling scenarios. The Runge–Kutta 4th Order (RK4) method delivers very high accuracy, but its multi-stage computation increases execution time and makes it computationally intensive, particularly in large-scale or real-time applications (Tay et al., 2015). The Euler method, while fast and simple to implement, can be inaccurate for stiff problems or when infiltration rates change rapidly, leading to significant numerical errors (Kong et al., 2021). The Heun method offers a balanced trade-off, combining moderate computational speed with good accuracy for smooth decay processes such as Horton's model (Pezeshk & Camp, 1994). The Adams–Bashforth scheme, on the other hand, is more efficient for long-term simulations but requires an initial warm-up period and can become unstable when faced with abrupt changes in infiltration dynamics (Rashidinia et al., 2018).

In this study, the evaluation of Euler, Heun, RK4, and Adams–Bashforth schemes is conducted entirely within a simulation-based framework, providing a controlled environment to quantify numerical accuracy and sensitivity without interference from measurement noise or site-specific uncertainties. While the present work focuses on an idealized Horton infiltration scenario, these numerical integration techniques are widely applied in real-world hydrological and environmental engineering problems. For example, Bayabil et al. (2019) applied infiltration modeling to support agricultural irrigation scheduling, optimizing water use efficiency based on soil infiltration capacity. Wang and Chu (2020) incorporated similar infiltration formulations into urban flood modeling systems, where accurate simulation of infiltration rates directly influences surface runoff prediction. Likewise, Godwin et al. (2022) used infiltration modeling to estimate aquifer recharge, integrating it with hydrogeological datasets to assess groundwater replenishment potential.

These studies demonstrate that the computational schemes assessed here have practical relevance beyond theoretical testing, serving as the numerical backbone for decision-support tools in water resource management, urban hydrology, and agricultural engineering. The comparative results from the present work thus provide a methodological basis for selecting an integration scheme that balances computational efficiency and accuracy in such applications, depending on the operational constraints and precision requirements of the modeling task.

Future Research

The current simulation-based assessment of numerical integration schemes for Horton's infiltration model can be expanded in several promising directions. First, the methodology will be extended to other widely used infiltration models, including the physically based Green–Ampt model, empirical formulations such as the Kostiakov model, and the Swartzendruber–Clague model, which is particularly relevant for permeable soils. Second, integration with machine learning approaches—such as Long Short-Term Memory (LSTM) networks or hybrid ML–numerical frameworks—has the potential to enhance predictive capability by combining process-based understanding with data-driven pattern recognition. Third, adaptive time-stepping solvers will be explored to improve computational efficiency by refining time steps during rapid infiltration changes and coarsening them when variations are minimal.

Field validation will be a priority in future work, using double-ring infiltrometers or TDR-based soil moisture sensors to collect infiltration data under varying soil textures (sandy, clayey, silty) and bulk densities. This will enable robust evaluation of model performance across different hydrological and land-use contexts. In addition, a parameter sensitivity analysis across diverse soil and moisture conditions will be incorporated better to understand the model's responsiveness to input variability. Finally, the use of high-performance computing, particularly GPU-accelerated RK4 simulations, will be investigated to facilitate large-scale, high-resolution infiltration modeling for applications in urban flood forecasting, precision irrigation management, and aquifer recharge assessment. These directions will address current limitations and move infiltration modeling from theoretical assessment toward robust, scalable, and field-validated decision-support tools.

5. Conclusions

This study compared four numerical methods for solving the Horton infiltration model, assessing both their accuracy and computational efficiency. The results showed that while the more complex methods (RK4 and Heun) provided higher accuracy, the differences in cumulative infiltration and recharge volume were not statistically significant, as confirmed by the ANOVA test. The Euler method, though the fastest, was less accurate but still provided reasonable estimates. Adams-Bashforth, with its efficient use of previous time steps, showed a good trade-off between speed and accuracy.

Euler's method is the fastest due to its simplicity — requiring only one function evaluation per step — but it has the lowest accuracy as a first-order method, with rapidly accumulating errors. Adams–Bashforth is also computationally efficient, especially for long simulations, requiring just one evaluation per step after initialization, but its second-order accuracy may degrade for problems with abrupt changes. Heun's method, with two evaluations per step, offers better stability and accuracy than Euler, making it a good balance between speed and precision. RK4, despite being the slowest with four evaluations per step, achieves fourth-order accuracy and is the most reliable for non-linear or stiff problems. Overall, the trade-off is clear: Euler and Adams–Bashforth favor speed, while Heun and especially RK4 prioritize accuracy. In practical applications, the choice of method should be guided by the specific requirements of the modeling task, balancing accuracy with available computational resources.

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